Unified Engineering Fall 2007
Fluids Problem F10

A Piper Cub is flying at a steady speed of $V_\infty = 30\,\mathrm{m/s}$ (67 mph), at some low altitude where the air density is $\rho = 1.2\,\mathrm{kg/m^3}$ and ambient pressure is $p_\infty = 100\,000\,\mathrm{Pa}$.

The figure below shows the velocity distribution $u(x)$ along a streamline just above the wing, as seen by the pilot. The flow direction is nearly along the x axis, so we will consider only the accelerations and velocities in the x direction, i.e. $v \simeq 0$ and $w \simeq 0$. To answer the questions below, and to give numerical results, use the local linear approximation to the velocity curve, shown as the heavy dashed line. Point A is at the midpoint of the 0.6 m interval shown.

1a) What is the *local acceleration* (or *local derivative*) of the fluid at point A? Explain.

1b) Estimate the *convective acceleration* (or *convective derivative*) of the fluid at point A.

1c) What is the true acceleration a_x that the air feels at point A?

2) Using your result from 1) in the x-momentum equation, determine the streamwise pressure gradient $\partial p/\partial x$ at point A.

3a) Using the Bernoulli equation together with the known freestream conditions, determine the pressure distribution $p(x)$ near A corresponding to the approximate linear velocity.

3b) Determine $\partial p/\partial x$ at point A from the Bernoulli-derived $p(x)$ in 3a). Do the x-momentum equation and Bernoulli equation appear to be consistent in this case?